Code No. 20MA1BS01 **R20**

TEEGALA KRISHNA REDDY ENGINEERING COLLEGE

(UGC – AUTONOMOUS)

B TECH I Semester Examinations, July 2021

(Common to CE, EEE, ECE, CSE, IT)

MATHEMATICS -I

Answer any Five questions

All questions carry equal marks

Time : 3 Hours **Max. Marks : 75**

1. a. If $A=\begin{bmatrix} -1 & 2+i & 5-3i \\ 2-i & 7 & 5i \\ 5+3i & -5i & 2 \end{bmatrix}$. Show that A is a Hermitian matrix and iA is skew Hermitian matrix.

b. Reduce the following matrix in to its normal form and hence find its rank

$$A=\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}.$$ [8+7]

2. a. Use Gauss Jordan method to find the inverse of the matrix $\begin{bmatrix} 8 & 4 & 3 \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix}$.

b. Solve the system of equations
$$\begin{aligned} 2x_1 + x_2 + 2x_3 + x_4 &= 6 \\ 6x_1 - 6x_2 + 6x_3 + 12x_4 &= 36 \\ 4x_1 + 3x_2 + 3x_3 - 3x_4 &= -1 \\ 2x_1 + 2x_2 - x_3 + x_4 &= 10 \end{aligned}$$ [7+8]

3. a. Find the Eigen values and Eigen vectors of the matrix $\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$.

b. Solve the following system of equation by Gauss Seidel method to obtain the final solution.

$$\begin{aligned} x + y + 54z &= 110 \\ 27x + 6y - z &= 85 \\ 6x + 15y + 2z &= 72 \end{aligned}$$. [8+7]

4. a. Verify Cayley Hamilton Theorem for the matrix $A=\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$.

b. Reduce the following quadratic form to canonical form by orthogonal transformation $2x_1^2+2x_2^2+2x_3^2+2x_1x_3$. [8+7]

5. a. Test the Convergence of the series. $\frac{1}{\sqrt{1}+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}+\ldots$

b. Discuss the convergence of the the series $\sum_{n=2}^{\infty}\frac{1}{n(\log n)^p}$. [8+7]

6. a. Discuss the convergence of the series $\sum\frac{\sqrt{n}}{\sqrt{n}+1}x^n$ by D'Alembert's ratio test.

b. Test whether the following series are absolutely convergent or conditionally convergent. $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{2n-1}$. [7+8]

7. a. Show that $\frac{b-a}{1+b^2}<\tan^{-1}b-\tan^{-1}a<\frac{b-a}{1+a^2}$ if $0<a<b$ and Deduce that $\frac{\pi}{4}+\frac{3}{25}<\tan^{-1}\frac{4}{3}<\frac{\pi}{4}+\frac{1}{6}$.

b. Prove that $\Gamma(m)\Gamma\left(m+\frac{1}{2}\right)=\frac{\sqrt{\pi}}{2^{2m-1}}\Gamma(2m)$. [8+7]

8. a. If $u=\log\left(x^3+y^3+z^3-3xyz\right)$, then prove that $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=\frac{3}{x+y+z}$ and hence show that $\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)^2 u=\frac{-9}{(x+y+z)^2}$.

b. Determine the maxima or minima of the function $\sin x+\sin y+\sin(x+y)$ [8+7]

---oo0oo---